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Exploring the Pedagogical Content Knowledge for Teaching Probability in Middle School: A South African Case Study

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ABSTRACT This paper reports on an exploration into in-service teachers' pedagogical content knowledge required for the delivery of lessons in probability. The section on probability is taught in Mathematical Literacy to the middle school learners in South Africa. As a theoretical framework the work initiated in the nineteen eighties was sought. This paper adopted a refined framework into domains of pedagogical knowledge. The four domains are: *Common Content Knowledge; Specialised Content Knowledge; Knowledge of Content and Students* as well as *Knowledge of Content and Teaching* which are defined and applied to show their necessity in a teaching situation. Examples of tasks from an open ended questionnaire were discussed and identification of the pedagogical content knowledge on probability was made. Teachers' written responses were collected and analyzed to verify or refute suggested strands of knowledge provided earlier. The written responses were from in-service teachers (n = 86) who were studying towards an Advanced Certificate in Education (an upgrading qualification) at a South African university.

INTRODUCTION

The concept of probability is significant in making learners understand their social environment and be able to calculate and determine various chances for specific events to occur or not. It is also a topic that is prescribed by the Department of Education in the Learning Programme Guidelines of the subject Mathematical Literacy. Learning Outcome 4 of the National Curriculum Statement states that:

"...the learner is able to collect, summarise, display and analyse data and apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions" (DoE 2008: 11).

Although in Mathematical Literacy the topic is not dealt with in a manner as detailed as in the core Mathematics, it nevertheless does give learners the necessary idea of how chance affects their everyday life decisions. A very sad discovery from the researcher's past experience though, is that most learners enter Grade 10 without any clear understanding of the notion of probability, when it is a concept that should have been introduced in the General Education and Training phase (GET). This topic is generally assigned a minimum of four lessons in a school with forty five minutes. This is excluding a test or tutorial on the section.

The Revised National Curriculum Statement Grades R-9 (schools) states that when learners in grade 8 having done Data Handling will pose questions relating to human rights, social, economic, environmental and political issues in their own environment. They will also be able to select appropriate sources for the collection of data. Wessels (2008) has stated that selected contexts, using discrete data involving only whole numbers, are used to build awareness of human rights and other social, economic and environment issues. He says that in this way a learner develops the ability to critically analyze data collection methods, interpretations and predictions from data.

On the other hand the teachers' guide for the development of learning programs in schools (DoE 2008) has stated that effective teaching relies on an understanding of mathematics and an understanding of what learners know, what they need to know and structuring learning opportunities appropriate to the needs of the particular learners that will support and encourage their teaching.

Therefore from the above discussion on the importance of statistics and probability in the learner's life and what the RNCS and Researchers has suggested about Data handling the researcher felt that the topic on data handling is relevant and fits with the RNCS and is appropriately placed in the school curriculum. Mosvold and Fauskanger (2014) emphasise that teachers play an important role as far as the quality of children's learning is concerned and teachers' knowledge is a factor of crucial importance. In their paper they present and discussed how teachers' discussions elicited beliefs about their knowledge to teach mathematics. Bataner et al. (2014) in their paper explored ways of building high school pre-service teachers knowledge to teach correlation and regression. The researcher in this paper shows the necessary content knowledge mathematics teachers require in probability as these are the foundational blocks upon which topics like regression and correlation can be built on.

According to Krauss et al. (2008) teachers need not only understand the mathematical concepts underlying the questions that learners ask but they also need to know how these concepts can best be explained. They further cite Ball et al. (2005) saying that it has been shown that a deep understanding of mathematical concepts may enable teachers to access a broad repertoire of strategies for explaining and representing mathematical content to their students.

The researcher decided to explore the necessary pedagogical content knowledge for teaching data handling and probability to middle school children in a South African context. This paper reports on this exploration and results from a call made by other research papers (Brijlall and Maharaj 2014; Bansilal et al. 2014). Brijlall and Maharaj (2014) focussed their work with high school mathematics teachers. They found that certain areas of the teachers required attention. They suggested other studies to address these shortcomings of teachers. Another paper, by the researchers Bansilal et al. (2014), also showed the disappointing outcomes of high school teacher preparedness in South Africa. These findings certainly suggest the need for further exploration in this area of PCK.

Theoretical Framework

In discussing the learning and teaching process, it should always be noted that teaching does not only involve a teachers' knowledge of the subject matter. Much research has been focused on teachers' knowledge of the content in a subject or lack thereof, in explaining the resulting learner achievement. However, Ball et al. (2008) insist that the organising principles and structures and the rules for establishing what is legitimate to do and say in a field must be understood by teachers. Teachers are not only expected to know that some concept is as it is, they should also know *why* it is as it is. This requirement brings the notion of pedagogical content knowledge into fore. The main purpose of pedagogical content knowledge is to bridge content knowledge with the practice of teaching. Hence this study made use of the theoretical framework of pedagogical content knowledge as outlined by Shulman (1986), Ball et al. (2008), Brijlall (2011) and Ozden (2008).

Various researchers like Shulman (1986, 1987) have made a huge contribution in trying to understand what pedagogical content knowledge entails. According to Shulman (1986), pedagogical content knowledge refers to the most useful ways of representing and formulating the subject that make teaching of that subject comprehensible to others. Other researchers have also presented their definitions. Niess (2005) as cited in Ball et al. (2008) defined pedagogical content knowledge as the intersection of knowledge of the subject with knowledge of teaching and learning. Lowery (2002: 69), also cited in Ball et al. (2008), referred to pedagogical content knowledge as "that domain of teachers' knowledge that combines subject matter knowledge and knowledge of pedagogy". With these various definitions, pedagogical content knowledge seems to be a product of changing the subject matter to what will be learnable to learners and thus creating an understanding of what is a difficult or an easy topic.

The argument arises as to where teachers are supposed to gain the knowledge of teaching, of defining concepts, of selecting relevant examples and exercises, of choosing the sequence in treating a specific topic and of distinguishing between wrong and correct strategies in solving problems (Brijlall et al. 2011). With research and experience teachers are expected to gain such competency although this expectation ignores the novice teacher. Textbooks themselves are not always reliable because they may present faulty information which a teacher is expected to pick up at the first instance. How then do we account for all the understanding and planning that we expect our teachers to have? Such demands in teaching are what made Ball et al. (2008) to come up with what they called the four domains of pedagogical content knowledge. These are: 1) Common Content Knowledge, (CCK), 2) Specialised Content Knowledge, (SCK), 3) Knowledge of Content and Students, (KCS) and 4) Knowledge of Content and Teaching (KCT). They defined these domains as follows:

Common Content Knowledge

The knowledge of mathematical content which is not only used in settings of teaching and learning. A mathematician who has studied some mathematics to a certain extent will have such knowledge. Teachers as well, should have CCK so that they readily recognise wrong answers provided by their learners; when textbooks give inaccurate definitions or have faulty information.

Specialised Content Knowledge

This type of content knowledge is the sole possession of teachers and its use is unique to teaching. Its use is only necessary for purposes of imparting mathematical content knowledge to others specifically. Teachers have to know various strategies of solving the same problem where this is the case. Their knowledge should be "beyond that of their students" (Ball et al. 2008: 399). They should make features of certain content easy to learn for their students.

Knowing Content and Students

As a teacher, one should know about both, ones' students and the mathematics. It should be known to the teacher what the learners will think and their reaction to specific information should be anticipated. A teacher should know the type of context that will be interesting and easy to understand to his/her learners. This includes the ability to choose tasks that students will find motivational as well as the ability to identify errors that will be common to his/her learners. Content knowledge is necessary but is not sufficient on its own (Brijlall 2011).

Knowing Content and Teaching

A teacher, to introduce a new concept, should know what the learners already know and must therefore be able to sequence her lessons in manner that is likely to enhance and facilitate learning of a new concept. Knowing the content and knowing how to teach helps the teacher to identify moments where learners' suggestions can be attended to or be rejected for a later stage. Brijlall (2011) and Brijlall et al. (2011) believe that a strong link exists between content knowledge and the practice of teaching. A teacher should not be distracted by a learner's interjections.

Ball et al. (2008) also mention what they call horizon knowledge and they assert that teachers should have a global picture of a concept that they teach. They should know about sections done in lower classes that will be needed in higher classes and beyond. The connection between mathematics topics for work that will come at a later stage should allow the teacher to set basic foundations. This displays knowledge of content and curriculum, and is very crucial especially for the lower grade teachers. With this in mind I formulated the research question: *What pedagogical content knowledge is required for teaching probability to middle school mathematics learners?* I am of the opinion that research and continued practice are vital sources of these various categories of knowledge which are undoubtedly a pre-requisite for any efficient learning and effective teaching process to take place.

METHODOLOGY

The researcher was granted permission to do this qualitative study at an educational faculty in a South African university. The in-service teachers who participated in this study were registered for an upgrading qualification in mathematical literacy. Although over five hundred teachers enrolled the researcher carried out the data capture with eighty- six of these teachers. It is argued that "Qualitative inquiry typically focuses on relatively small samples, selected *purposefully* to permit inquiry into and understanding of phenomenon in *depth*" (Cohen et al. 2007).

These in-service teachers were provided with an open-ended questionnaire to complete. The first item on this open-ended questionnaire required the participants to sequence four lessons on the topic of probability which is taught to grade nine (fourteen/fifteen year old) children. The other six questionnaire items were on problem solving, dealing with concepts based on probability tasks which are relevant to the teaching at this grade. The researcher analysed some of the written tasks and identified and extracted the pedagogical content knowledge from the written responses of these teachers. Despite the focus being on the extraction of the pedagogical content knowledge (PCK) necessary for the teaching of probability the researcher noted that the teachers performed satisfactorily in the data handling module. Sixty- three of these eighty-six teachers had scored a mark over seventy percent. The assessment task dealt with central tendencies and probability problems. The researcher therefore thought that the responses provided by these teachers were reliable when gauging their suggestions on what pedagogical content knowledge was necessary for the section of probability when teaching this section to children in the middle schools in South Africa.

ANALYSIS AND DISCUSSION

The researcher commenced with the data capture of item one. This dealt with the sequencing of lessons. Table 1 illustrates the sequencing of the lessons as derived from the data capture.

Table 1: The sequencing of lessons made by the in-service teachers

Lesson	*Concept/ probability task*	*Number of responses making this choice*
One	Definition of probability(with tasks based on this definition)	78 (62)
Two	Probability scale (with tasks based on this scale)	52 (48)
Three/Four	Problem solving in probability	63

This table shows the sequencing of the lessons based on the majority response. An overwhelming number of teachers (78) indicated that the first lesson on probability should be based on the definition of probability as required for this grade of learners. Not all seventy- eight of them mentioned tasks based on this definition. Only sixty- two of these in-service teachers also made some mention of this. Most South African mathematical literacy textbooks define probability as a positive number less than or equal to one. Probability is defined as:

$$\text{Probability} = \frac{\textit{the frequency of a specific event}}{\textit{total number of possible outcomes of that event}}$$

For grade 10 learners, this is expected to become prior knowledge. However, as a teacher who has content knowledge of the subject and its organising structures (Ball et al. 2008), one does not assume that learners are familiar with any part of the content without actually finding out through much simpler tasks and questions. Hereafter, suitable tasks on this definition could follow as displayed by the responses of the sixty- two in-service teachers (see Table 1). The tasks suitability could be elicited by incorporating explanations by learners to the meaning of the number obtained after using the definition. For example if children got an answer of one in a task, the teacher should ask for the implication of such an outcome. This type of reasoning would prepare the learners for the next lesson based on the probability scale (see Fig. 1). Fifty-two of the eighty- six in-service teachers agreed that the second lesson should deal with the probability scale (see Table 1). As stated by Ball et al. (2008), one of their four domains of teaching is what they call *knowledge of content and teaching*. It is for this reason that before learners can attempt the first two lessons the teacher should revise the learners' knowledge of fractions. Learners have to know how to convert from decimal to common, from common to percentage, from decimal to percentage and vice versa (Thembela 2011). As a teacher one should be in a position to sequence content for instruction where one chooses where to start and which examples to use in an attempt to "take learners deeper into the content" (Ball et al. 2008: 401). The probability scale is what looks as follows:

I———————————————————I

0	0.5	1
Impossible	Fifty-fiftychance	Certain

Fig. 1. The probability scale taught to grade nine learners

The terminology of *less-likely; more-likely; highly-likely* is introduced after learners have mastered the idea that as the probability approaches zero (0), the less likely the chances become of an event taking place and as the probability approaches 1, the more likely that an event will occur. It is also at this stage that the *common content knowledge* of fractions of the learners is put to test. Undoubtedly, learners at first glance are not likely to recognise that there are an infinite number of fractions between 0 and 0.5 as well as between 0.5 and 1. The teacher, with his/her *specialised content knowledge* should therefore test learners' knowledge by

using various fractions such as $\frac{31}{107}, \frac{1250}{5000}, \frac{2}{14379}$, etc. In this way the teacher would be making sure that learners realise the infinite number of fractions that can fit within the probability scale. Concepts of the simplification of fractions like to will also crop in at this stage. However, this will all depend on the teachers' specialised content knowledge where he/she knows what examples to use, how to use them and for which purpose these are used. Therefore it is imperative that the teachers' knowledge of content and teaching be put into correct practice (Thembela 2011). Shulman (1986), referred to this knowledge as pedagogical content knowledge when he says that this knowledge " goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching*" (Shulman 1986: 9). He further added that analogies, illustrations, examples and demonstrations that make the subject comprehensible to others are included in the pedagogical content knowledge. As a teacher, it becomes his/her duty to ensure that learners realise that to be able to express any probability, one has to know all the possible outcomes first. This can only be achieved depending on the examples that are provided by the teacher. For lessons three and four, the majority of these in-service teachers felt that problem tasks based on probability tasks as expected by curriculum policy documents (CAPS 2011) should be facilitated with learners.

In order to gather more information on the pedagogical content knowledge for probability the researcher considered the written response of IT41 on a problem providing information in tabular form. The researcher looked at the response to item 2.2 from the questionnaire. The written response to task 2.2.1 is shown in Figure 2 and item 2.2.2 in Figure 3.

In Figure 2, it can be observed that the teacher could successfully find the missing data as desired by the problem statement. The pedagogical knowledge displayed here is one which is instrumental as it involved arithmetical operations. This type of knowledge could fall in the domain CCK since it deals with the type of context not only used in the setting of teaching and learning. For example, a similar kind of reasoning would prevail if one was working with a puzzle. However, for the second part of the task, the teacher displayed conceptual understanding. He needed to acknowledge understanding of the concept of "independent event". This understanding is evident in the manner he presented his solution. In Brijlall (2011), Adler et al. (2002) are cited as saying that conceptual knowledge

2.2. A survey was conducted about a particular breakfast cereal. 200 males and 160 females were interviewed. The table below shows some of the results.

	Males	Females	Total
Liked the cereal	(a) 120	35	155
Did not like the cereal	80	(b) 125	(c) 205
	200	160	(d) 360

2.2.1. Calculate the values of the letters (a, b, c and d) in the table. (4)

(a) = 120
(b) = 125
(c) = 205
(d) = 360

4

Fig. 2. An extract of the teacher's response to the first part of the task

is a special way in which "a teacher uses the mathematical content in order to teach mathematics" (Brijlall 2011: 36). A difference is noted by Adler et al. (2002) in the way in which general mathematicians view the mathematical content to the way in which mathematics teachers view the content. For self-explanatory reasons, knowledge of content to mathematician is sufficient for own use whereas a mathematics teacher will always have the welfare of the learners at heart. The teacher of mathematics may stand as an expert in the subject, but he always has to lower his standard to those of the learners. His/her ultimate goal is to impart the knowledge in the best possible way taking into consideration the knowledge and position of the learner. The type of knowledge illustrated in Figure 3 could fall in the domain Specialised Content Knowledge (SCK). We note also the understanding of multiplication of decimal numbers is also vital for effective answering of the item task.

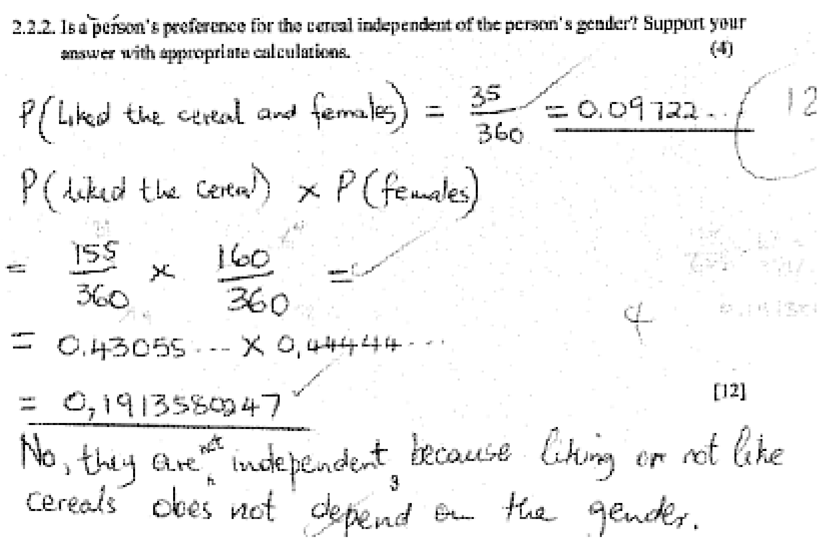
Figure 4 explicitly displays that the in-service teacher IT41 who performed well in item 2 performed dismally in the questionnaire item 5. Here he showed two weaknesses. One, he demonstrated the wrong number of "ACES" in a pack of cards. Secondly, he does not extend the concept of "both" in items 5.1 and 5.3. It seemed that IT41 used the denominator after subtracting the two cards removed (see Fig. 4). This could mean that he did not understand the meaning of the denominator in the definition of the probability concept.

Ball et al. (2008) further assert that over and above the different types or domains of knowledge that have been discussed, there exists another category which they call the *horizontal knowledge*. This category emphasizes that a teacher must be aware of how mathematical topics are related over the span of mathematics that is included in the curriculum. It is important for these students to use language as a tool to help develop their mathematical reasoning. This is in keeping with the findings of Molefe et al. (2010). The following example highlights the use of language:

Your sister has children. What is the probability that your sister has:

i. boys only?
ii. a boy, a girl and another boy?
iii. two boys and then a girl?
iv. two boys and a girl?
v. a girl as a first born?

What should be noted from this example is the way in which a misunderstanding of the English phrase in the question may lead to an incorrect response. The teacher would have given such questions on purpose because he/she has the knowledge of the content and the students. For example, learners who are English second language speakers may encounter a problem in trying to identify the difference in ques-


2.2.2. Is a person's preference for the cereal independent of the person's gender? Support your answer with appropriate calculations. (4)

P(Liked the cereal and females) = 35/360 = 0.09722... 12

P(liked the cereal) × P(females)

= 155/360 × 160/360 =

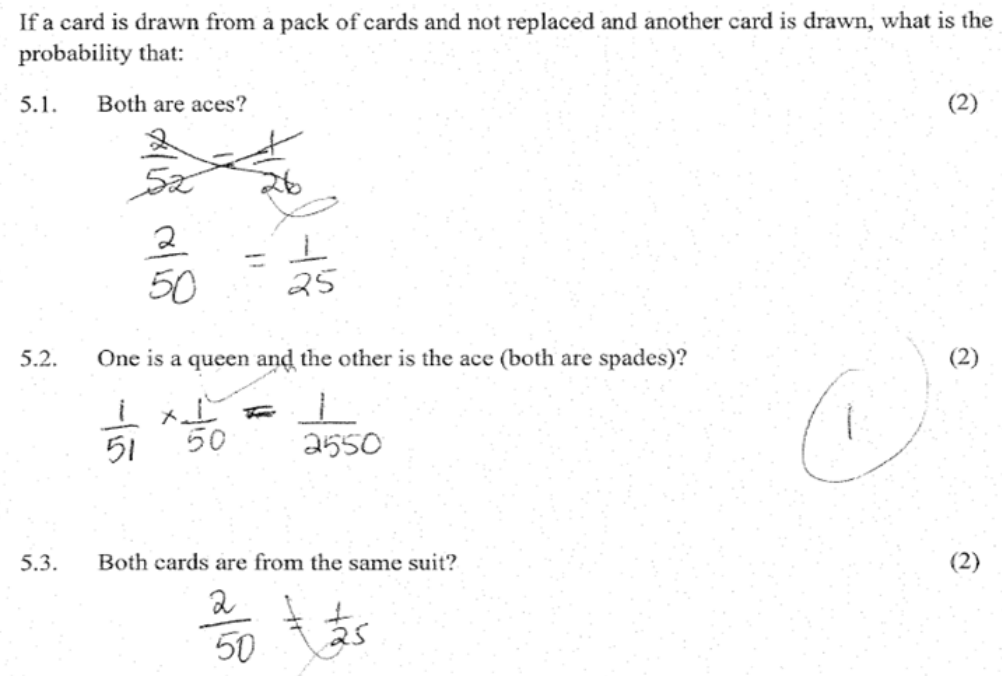
= 0,43055... × 0,44444...

= 0,1913580247

[12]

No, they are not independent because liking or not like cereals does not depend on the gender.

Fig. 3. The teacher's response to the second part of the task



If a card is drawn from a pack of cards and not replaced and another card is drawn, what is the probability that:

5.1. Both are aces? (2)

$\cancel{\frac{2}{52} = \frac{1}{26}}$

$\frac{2}{50} = \frac{1}{25}$

5.2. One is a queen and the other is the ace (both are spades)? (2)

$\frac{1}{51} \times \frac{1}{50} = \frac{1}{2550}$

1

5.3. Both cards are from the same suit? (2)

$\frac{2}{50} = \frac{1}{25}$

Fig. 4. The written response by IT 41 to item 5

tions (ii); (iii) and (iv). In fact, not only second language speakers, but first language speakers as well may not notice the difference brought about by the use of the word "then" in number (iii). The teacher could also say that answers should be given in decimal form or as a percentage. That again would be a teacher showing his conceptual and horizontal knowledge of the topic and the subject.

Other examples may be given to learners where they will make use of the tree diagram or contingency tables. Again, it is the teacher with his specialised content knowledge and knowledge of content and his students together with knowledge of content and teaching that will play a pivotal role in the selection of relevant exercises (Thembela 2011).

CONCLUSION

The teaching process involves more than just the teacher's knowledge of the content in a specific subject. Teaching is not only about a teacher having the necessary knowledge of the subject matter. It does not only involve knowledge of mathematics by the teacher. For the teaching of probability we found that the three domains Common Content Knowledge (CCK), specialised content knowledge (SCK) and Knowledge of Content and Teaching (KCT) featured when identifying the pedagogical content knowledge necessary for teaching probability. The CCK was identified when analysing the written response of IT41. Also, the SCK included knowledge of: 1) definition of probability, 2) recall of the probability scale and attaching meaning to a number on this scale, 3) thorough understanding of fractions and their operations and 4) translating correct English vocabulary usage into mathematical notions relevant to probability tasks.

As this was a small scale study, further research is necessary to try and accommodate a greater number of teachers in the process of acquiring the different categories of content knowledge that are necessary for the teaching and learning process in other topics of the mathematics curriculum. This study makes a contribution to the theory of the domains of PCK and the didactics of probability at middle school.

RECOMMENDATIONS

Pedagogical content knowledge plays a crucial role in making the teaching process more application based and meaningful. It is not adequate to perceive pedagogical content knowledge as the strategies used in teaching and classroom administration methods only. Common content knowledge of a subject may be known to any mathematician, generally stands different from specialised content knowledge that happens to be a unique attribute of teachers. Thus, it is recommended for authorities responsible for teachers' development to keep in mind that teachers further require knowledge of the content and students that they teach. This helps them in making correct choices and taking the right decisions about the mathematics to be learnt by their learners. Over and above these domains, teachers need to have knowledge of both content and teaching, so that they may be enabled to structure the teaching process in a sequential manner. Finally, teachers have to consider not only what they teach in a specific grade. They have to know what will be learned in grades higher than the ones they are teaching. That is termed knowledge of content and curriculum.

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